

# LEVEL-SET BASED GEOMETRIC COLOUR SNAKE WITH REGION SUPPORT

*Xianghua Xie and Majid Mirmehdi*

Department of Computer Science,  
University of Bristol, Bristol BS8 1UB, England  
{xie, majid}@cs.bris.ac.uk

## ABSTRACT

A novel method is introduced to force a geometric-based snake be more tolerant towards weak edges and noise in images. The method integrates gradient flow forces with region constraints obtained from diffused region segmentation forces. The diffusion is obtained from the region map vector flow field. This extra region force gives the snake a global view of the boundary information within the image. We present results on both graylevel and colour images.

## 1. INTRODUCTION

Snakes are widely used in many applications, including segmentation, shape recovery, and motion tracking. Different models have been proposed which can be classified into two types: parametric snakes and geometric snakes. Parametric snakes [1] are maintained by a spline, explicitly represented as parameterised curves in a Lagrangian formulation while Geometric snakes [2, 3] are represented implicitly and evolve according to an Eulerian formulation [4]. They are based on the theory of curve evolution implemented via level set algorithms and do not need to reparameterise the curve or to explicitly handle topological changes. They are completely intrinsic and automatically handle changes in topology when numerically implemented using level sets. Hence, without resorting to dedicated contour tracking, unknown numbers of multiple objects can be detected simultaneously. Furthermore, geometric snakes can have much larger capture areas than parametric snakes.

However, geometric snakes are still not perfect with the major problem being that they suffer from leakage at weak edges into neighbouring image regions. There have been a number of improvements to the overall geometric snake performance, such as area-length minimization snakes [5], generalized gradient vector flow snakes [6], and region-based snakes [7]. Siddiqi et al. [5] could improve the performance of their snake by a linear combination of weighted length and weighted area gradient flows, but still did not provide a satisfactory solution to the weak-edge leakage problem [5, 4]. Xu and Prince [6] extended their gradient vector flow (GVF) snake into the Generalised GVF snake (the GGVF).

The gradient vector flow is useful when there are boundary gaps, because it preserves the perceptual edge property of snakes [6, 4]. However, the GGVF still has topological problems. Also when a weak edge lies beside a strong edge, the snake is likely to step through towards the strong edge. Region-based snakes rely on the segmentation of the image into regions. Chakraborty et al. [8] proposed a method to integrate the *parametric* snake with region segmentation. It requires apriori knowledge of the region of interest, and can suffer from topological problems. Geometric snake region-based methods have rarely been proposed, but a good example is [7]. The Geodesic Active Region model in [7] unifies boundary and region-based forces towards the region boundary by integrating into the geodesic snake framework a boundary/region segmentation technique based on modelling the image histogram using a mixture of Gaussians.

In this paper, a new region-based method is proposed to make the geometric snake much more tolerant to image noise and weak edges. We refer to it as the Region-aided Geometric Snake as it integrates gradient flow forces with diffused region forces. The diffused region force gives the snake a global view of the object boundaries. The theory is independent of any particular region force which in turn can be generated using different techniques according to the needs of the application at hand. Using colour edge gradients, the region-aided snake will be shown to naturally extend to object detection in colour images.

## 2. THE GEOMETRIC SNAKE

Geometric active contours are based on the theory of curve evolution and implemented numerically by employing level sets. Using a reaction-diffusion model from mathematical physics a planar contour is evolved with a velocity in the direction normal to the curve. The velocity contains two terms: a constant motion term that leads to the formation of shocks from which a representation of shape can be derived, and a curvature term that smooths the front, showing up significant features and shortening the curve. We now briefly review the formulation of the geodesic active contour [2, 3], hereafter also referred to as the standard geometric model.

Let  $C(x, t)$  be a two-dimensional active contour. The original formulation of the geometric model is given by

$$\frac{\partial C}{\partial t} = g(|\nabla I|)(\kappa + c)\vec{N}, \quad (1)$$

where  $t$  denotes the time,  $\kappa$  is the Euclidean curvature,  $c$  is a constant value, and  $\vec{N}$  is the unit inward normal of the contour.  $g(\cdot)$  is a positive, decreasing weighting function such that  $g(x) \rightarrow 0$  as  $x \rightarrow \infty$ , and  $g(x) \rightarrow 1$  as  $x \rightarrow 0$ . In application to shape modelling the weighting factor could be an edge indication function, which has larger values in homogeneous regions and very small values on the edges. This model performs well in practice for objects with clear, good contrast. However, with gaps or indistinct parts on the edges, the snake will easily leak through. Gaps may also remain between the converged snake and the true boundary. Hence, the following improvement was proposed [2]:

$$\frac{\partial C}{\partial t} = g(|\nabla I|)(\kappa + c)\vec{N} - (\nabla g(|\nabla I|) \cdot \vec{N})\vec{N}. \quad (2)$$

The second term of (2) acts like a doublet, which attracts the active contour to the feature of interest since the vectors of  $\nabla g$  point toward the middle of the boundaries. For an ideal edge,  $g(\cdot)$  tends to zero. Thus it tries to force the curve to stop at the edge. However, the convergence quality still highly depends on the stopping term  $g(\cdot)$ . If  $g(\cdot)$  is not small enough along edges, there will be an underlying constant force mainly caused by the  $c$  term.

The geodesic or geometric active contour is numerically implemented using level set techniques [2, 3, 9].

### 3. REGION-AIDED GEOMETRIC SNAKE

Here we propose a novel approach to make the geometric snake much more tolerant towards weak edges. It comprises the integration of the gradient flow forces with diffused region forces in the image resulting in the *Region-aided Geometric Snake*. The gradient flow force supplants the snake with local object boundary information. The region force is based on the global features in the image, and acts as an extra image constraint, pulling the snake close to the region boundary. When reaching the vicinity of the region boundary, the proposed snake will rest on the local maximum based on the gradient flow forces. The steady state of the snake is the equilibrium state of the gradient forces and region forces. We show that this combination of forces not only improves the performance of the geometric snake toward weak edges, but also makes it more immune to noise.

The region force can be generated from any image segmentation technique, e.g. colour segmentation such as [10]. This means that while our proposed method is independent of any particular segmentation technique, it is dependent on the quality of the regions produced. However, we show, in

the limited space available here, a good degree of tolerance to (reasonable) segmentation quality, and that our snake can indeed also act as a refinement of the results of the earlier region segmentation. To demonstrate this, we examine both under-segmentation and over-segmentation options of the software from [10].

The segmentation splits the image into several regions giving the region map  $R$  with larger values at the region boundaries. Then we compute the gradient of this region map giving region constraints in the vicinity of the region boundaries. While the snake evolves in a homogeneous region, it does so mainly based on the gradient flow force. If the snake tries to step from one region into another, it must concur with the region force since it breaks the region criteria, which probably indicates a leakage. The capture area of the region force is quite small. The gradient vector diffusion method proposed in [6] is used to diffuse the region forces along the region boundaries. This then gives the region force a much larger capture area. We can state the solution of the generalized gradient vector flow equation on region map  $R$  as the equilibrium state of

$$\begin{cases} p(|\nabla R|)\nabla^2 u - q(|\nabla R|)(u - \nabla R_u) = 0 \\ p(|\nabla R|)\nabla^2 v - q(|\nabla R|)(v - \nabla R_v) = 0 \end{cases} \quad (3)$$

where  $\nabla^2$  is the Laplacian operator with dimensions  $u$  and  $v$ , and  $p(\cdot)$  and  $q(\cdot)$  are weighting functions that control the amount of diffusion and are selected such that  $q(\cdot)$  gets smaller as  $p(\cdot)$  becomes larger. A desirable result of this is that in the proximity of large gradients, there will be very little smoothing, and the vector field will be nearly equal to the gradient of the region map. We use the following functions for diffusing the region gradient vectors:  $p(|\nabla R|) = e^{-(|\nabla R|/K)}$  and  $q(|\nabla R|) = 1 - p(|\nabla R|)$  where  $K$  is a constant and acts as a trade-off between field smoothness and gradient conformity.

Now we can derive the proposed region-aided geometric snake formulation. The region force is treated as an extra external force of the snake. The original internal and external forces of (2) are given by

$$\begin{cases} F_{int} = g(|\nabla I|)\kappa\vec{N} \\ F_{ext} = g(|\nabla I|)c\vec{N} - \nabla g(|\nabla I|) \end{cases} \quad (4)$$

where  $g$  is the stopping function. Now we add the diffused region force into the external term:

$$\begin{cases} F_{int} = g(|\nabla I|)\kappa\vec{N} \\ F_{ext} = \alpha g(|\nabla I|)\vec{N} + \beta \vec{R} - \nabla g(|\nabla I|) \end{cases} \quad (5)$$

where  $\vec{R}$  is the region force vector field obtained in (3) and  $\alpha$  is a new constant incorporating  $c$ . Constants  $\alpha$  and  $\beta$  act as a trade-off between gradient forces and region forces. Finally, the region-aided geometric snake formulation is:

$$\frac{\partial C}{\partial t} = [g(|\nabla I|)(\kappa + \alpha) - \nabla g(|\nabla I|) \cdot \vec{N} + \beta \vec{R} \cdot \vec{N}]\vec{N}. \quad (6)$$

Using Osher and Sethian [9] as the basis for the numerical algorithm for curve evolution, the level set representation of the region-aided snake in (6) can be shown as:

$$\frac{\partial \phi}{\partial t} = g(|\nabla I|)(\kappa + \alpha)|\nabla \phi| + \nabla g(|\nabla I|) \cdot \nabla \phi - \beta \tilde{R} \cdot \nabla \phi \quad (7)$$

The curvature is given by  $\kappa = \nabla \frac{\nabla \phi}{|\nabla \phi|}$ .

#### 4. REGION-AIDED COLOUR SNAKE

As shown in [2], the theory of boundary detection by the geometric (geodesic) snake can be applied to any general 'edge detector' function. The stopping function  $g(\cdot)$  should tend to zero when reaching the edges. Let  $f$  be the edge detector, then the decreasing function can be  $g = \frac{1}{1+f}$ . When dealing with graylevel images, the solution for  $f$  is quite straight forward, e.g.  $f = |\nabla(Gauss * I)|$ . We use a similar stopping function for edges obtained directly from colour images.

We compute gradients in colour images by following [11]. Let  $\Theta(x_1, x_2) : R^2 \rightarrow R^m$  be an image of  $m$  bands. The infinitely-small difference between two points becomes the arc element  $d\Theta = \sum_{i=1}^2 (\partial\Theta/\partial u_i) du_i$  and its squared norm is given by  $d\Theta^2 = \sum_{i=1}^2 \sum_{j=1}^2 \frac{\partial\Theta}{\partial u_i} \frac{\partial\Theta}{\partial u_j} du_i du_j$ . Using standard notation of Riemannian geometry, we have

$$d\Theta^2 = \begin{bmatrix} du_1 \\ du_2 \end{bmatrix}^T \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} du_1 \\ du_2 \end{bmatrix}. \quad (8)$$

where  $s_{ij} = (\partial\Theta/\partial u_i) \cdot (\partial\Theta/\partial u_j)$ . The extrema of the quadratic form are in the directions of the eigenvectors of the metric tensor  $s_{ij}$ , with corresponding eigenvalues:

$$\lambda_{\pm} = \frac{s_{11} + s_{22} \pm \sqrt{(s_{11} - s_{22})^2 + 4s_{12}^2}}{2}. \quad (9)$$

The eigenvalues provide the maximal and minimal changes at a given point in image. Thus the vector edges are expressed by how  $\lambda_+$  compares to  $\lambda_-$ . Then, if we let  $f_{col} = (\lambda_+ - \lambda_-)$  define the edges, the stopping function can be written as  $g_{col} = \frac{1}{1+f_{col}}$ . Substituting  $g_{col}$  for the stopping function  $g$  in (6), we can extend the region-aided snake to a colour region-aided snake:

$$\frac{\partial C}{\partial t} = [g_{col}(|\nabla I|)(\kappa + \alpha) - \nabla g_{col}(|\nabla I|) \cdot \tilde{N} + \beta \tilde{R} \cdot \tilde{N}] \tilde{N}. \quad (10)$$

The corresponding substitution in (7) will lead to the corresponding level set implementation.

#### 5. IMPLEMENTATION AND RESULTS

The diffused region constraints can be generated in a variety of ways. In our experiments, we used the mean shift algorithm [10] to perform the initial region segmentation. The diffused state was then generated according to (3).

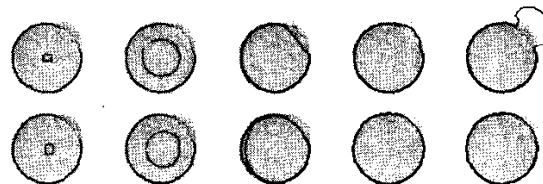


Fig. 1. Weak-edge leakage - Top: geodesic snake steps through; Bottom: proposed snake rests using extra region force.

#### 5.1. Preventing Weak-edge Leakage

We demonstrate the weak-edge leakage on a synthetic image (as in [5, 4]). The image contains a circular shape with a small blurred area on its boundary. The geodesic snake stepped through the weak edge because the intensity changes so gradually that there is no clear boundary indication in the edge map. Region-aided snake converged to the boundary since the extra diffused region force prevented the snake from stepping through (see Fig. 1) as it delivers useful global information about the object boundary.

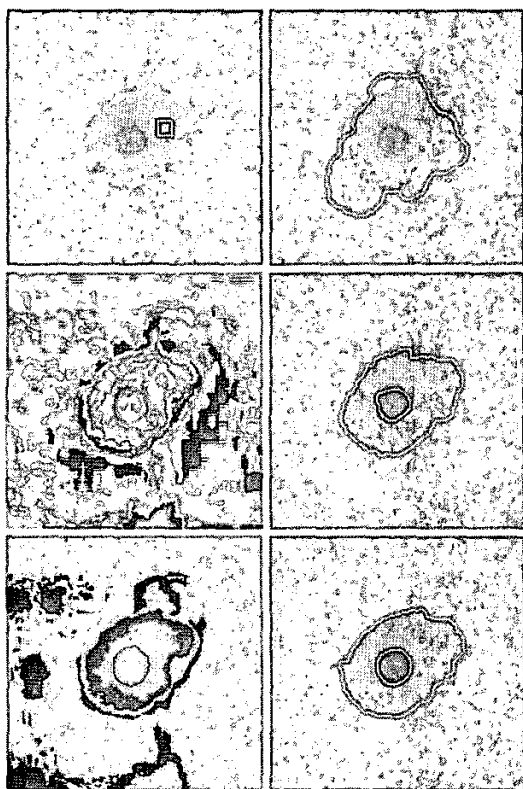
Fig. 2 shows the results on a colour image of a bacteria cell with both strong and fuzzy region boundaries. This is hard for the geometric snake because the weak edges are difficult to detect without global information. They dilute gradually into the background in certain places while strong edges occur inside and outside the cell as well. The region-aided snake (last two rows of Fig. 2) reaches its steady state and successfully converges to the cell boundary irrespective of whether the under or over-segmentation option in [10] is used to generate the region map (as shown).

In Fig. 3 a close up view of a retinal disc is shown as another example. The boundary of the optic disk is quite fuzzy and blended with the background. Again, the diffused region force helps the proposed snake stop at weak edges. Further images and experiments can be found online<sup>1</sup>.

#### 5.2. Testing on Noisy Images

We also tested the region-aided snake for tolerance to noise. Fig. 4 presents an example shape with a Harmonic curve boundary along with 20, 40, and 60% noise added to the original image. We also show the (pre-diffusion) region map in each case followed by the geodesic snake and the region-aided snake. The results clearly demonstrate the superior segmentation quality of the proposed snake. At low noise, both methods can find the boundary accurately. However, as the noise increases, more and more local maximums appear in the gradient flow force field which prevents the geodesic snakes from converging to the true boundaries. On the other hand, the region-aided snake has a global view of the noisy image due to the diffused region flow forces which pushes the snake towards the boundary.

<sup>1</sup><http://www.cs.bris.ac.uk/home/xie/ragss.html>



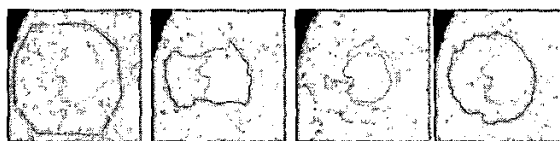
**Fig. 2.** Weak edge leakage - top: starting contour and geodesic snake (steps through), middle: over-segmented colour region map and converged region-aided snake, bottom: under-segmented colour region map and converged region-aided snake.

## 6. CONCLUSIONS

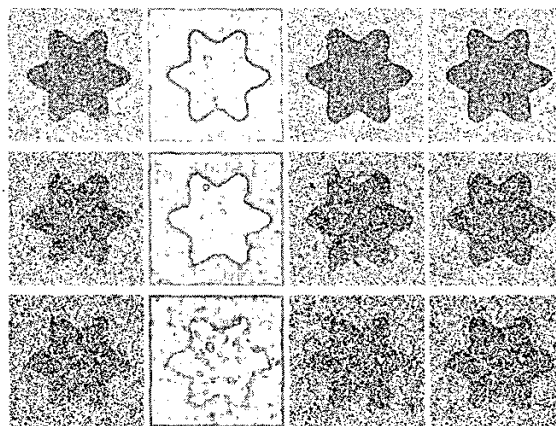
A novel region-aided geometric snake was introduced which integrates gradient flow forces with region constraints. The region force can be generated from any region segmentation technique, followed by gradient vector diffusion of the region map. The experimental results show that the region-aided snake is much more robust toward weak edges as well as more tolerable to noise. We demonstrated the application of the region-aided snake to colour images.

## 7. REFERENCES

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**Fig. 3.** Optic disk localisation, from left: initial state, final geodesic snake, region map, final region-aided snake.



**Fig. 4.** Shape recovery noise analysis for 20, 40, and 60% noise (on each row from left): image with added noise, region map, converged geodesic snake, and converged region-aided snake.

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